A SIMPLIFIED MODEL OF URBAN RAILWAY SYSTEM FOR DYNAMIC TRAFFIC ASSIGNMENT

T. SEO [a], K. WADA [b] and D. FUKUDA [c]
[a] *Department of Civil and Environmental Engineering, School of Environment and Society, Tokyo Institute of Technology, Japan*
Email: t.seo@plan.cv.titech.ac.jp
[b] *Institute of Industrial Science, The University of Tokyo, Japan*
Email: wadaken@iis.u-tokyo.ac.jp
[c] *Department of Civil and Environmental Engineering, School of Environment and Society, Tokyo Institute of Technology, Japan*
Email: fukuda@plan.cv.titech.ac.jp

ABSTRACT

Urban mass transit such as metro plays a significant role in transportation in metropolitan areas. Its most notable usage is the morning commute situation. It is known that such transit can be delayed due to congestion under excessive passenger demand. However, there are no tractable models of transit systems considering physics of its delay and congestion. It makes difficult to obtain realistic policy implications on morning commute with excessive demand. In this study, in order to account this issue, an analytical model of dynamic urban rail transit is developed considering physical interaction between the train-congestion and passenger-congestion. The model is based on fundamental diagram of rail transit system which is analytically derived by this study, and an exit-flow model. The notable feature of the proposed model is its high analytical tractability so that it can be applied to obtain policy implications on morning commute problems.

Keywords: (up to five): urban rail mass transit; dynamic traffic assignment; traffic flow theory

1. INTRODUCTION

Urban mass transit such as metro plays a significant role in transportation in metropolitan areas. Its most notable usage is the morning commute situation, in which excessive passenger demand is generated during a short time period. Several studies (e.g., Tabuchi, 1993; Kraus and Yoshida, 2002; Tian et al., 2007; de Palma et al., 2015) have analyzed such situations with some simplifications in order to obtain policy implications on management strategies for them (e.g., pricing and control in the morning commute problems).

It is known that urban mass transit often suffered from delay caused by congestion, even if there are no serious incidents nor accidents (Kato et al., 2012; Tirachini et al., 2013; Kariyazaki et al., 2015). For example, following two types of congestion are observed in rail transit:

- *train-congestion*: congestions involving consecutive trains using the same tracks.
- *passenger-congestion*: congestion of passengers at station platforms, namely, passenger's bottleneck congestion at the boarding doors of a train while it is stopped at a station.

These two types of congestions interact with each other (Wada et al., 2012; Tirachini et al., 2013). For example, passenger-congestion can extend dwell time of a train at a station. Then, the unintentionally extended dwell time interrupts the operation of the following train and causes train-congestion due to its high service frequency. Since service frequency is deteriorated by the train-congestion, the passenger-congestion at stations intensifies—this is a vicious cycle for a short term. In addition, for a long term, observational studies show that passengers in the Tokyo metropolitan area suffer such train delay and congestion almost daily (Kariyazaki et al., 2015); and their choice of departure time is affected by the chronic delays caused by the congestions (Kato et al., 2012). The various effects of these congestions are comprehensively reviewed by Tirachini et al. (2013).

Therefore, in order to obtain realistic policy implications on morning commute with excessive demand (which is the case for many metropolitan areas), dynamics of transit systems including delay and congestion must be taken into account, just like similar problems in road traffic (Arnott et al., 1993). However, to the authors' knowledge, no study has investigated the morning commute problems in transit systems with dynamic delay and congestion—in the aforementioned studies (Tabuchi, 1993; Kraus and Yoshida, 2002; Tian et al., 2007; de Palma et al., 2015), travel time of transit system is assumed to be constant and/or determined by static models. This might be due to that we do not have tractable models of transit systems considering physics of its dynamic delay and congestion.

The aim of this study is to develop an analytical model of the dynamics of an urban rail transit considering physical interaction between the train-congestion and passenger-congestion, while keeping its analytical tractability high so that it can be applied to obtain policy implications on morning commute problems. In Section 2, a simple and tractable operation model of rail transit is formulated that considers train-congestion, passenger-congestion, and the interaction between them. The model describes theoretical relation between passenger-flow and speed under ideal conditions—that is, a fundamental diagram. Then, in Section 3, a macroscopic and dynamic model of rail transit is developed by extensively employing a continuous approximation approach with the fundamental diagram, which is also widely used for auto traffic flow—that is, an exit-flow model. Finally, in Section 4, the approximation accuracy of the macroscopic model is validated through a comparison with microscopic simulation.

2. FUNDAMENTAL DIAGRAM OF RAILWAY OPERATION

In this section, we analytically derive a fundamental diagram of an urban rail transit operation, namely, relation among train-flow, train-density, and passenger-flow, based on microscopic operation principles.

2.1 Assumptions on Railway Operation

We assume following principles on urban rail transit operation. They are twofold: train's dwell behavior at a station for passenger boarding and cruising behavior between stations. Note that they are equivalent to those employed by Wada et al. (2012).

The passenger boarding time is modeled using a bottleneck model. That is, the flow-rate of passenger boarding is assumed to be constant, μ_p, if there is a queue; and there is a buffer time (e.g., time required for door opening/closing), g_b, for the dwell time. Then, the dwell time of a train at a station, t_b, can be represented as

$$t_b = \frac{n_p}{\mu_p} + g_b, \tag{1}$$

where n_p is number of waiting passengers at the station (or total number of passengers who are getting in and off the train). Passengers waiting a train at a station are assumed to board the first train arrived—it means that passenger storage capacity of a train is assumed to be unlimited.

The cruising behavior of a train is modeled using the Newell's simplified car-following model (Newell, 2002). In this model, a vehicle travels as fast as possible while maintaining the minimum safety clearance. Specifically, $x_m(t)$, position of a train m at time t, is described as

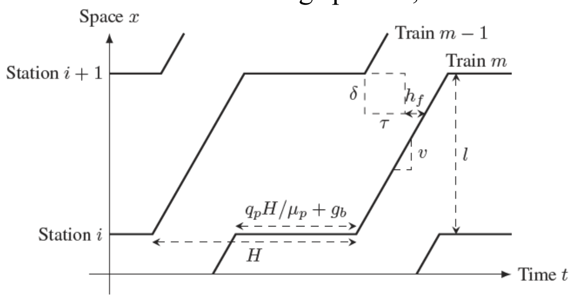
$$x_m(t) = \min\{x_m(t-\tau) + v_f\tau, x_{m-1}(t-\tau) - \delta\}, \tag{2}$$

where $m-1$ indicates the preceding train of train m, τ is the physical minimum headway time, v_f is the free-flow speed (i.e., maximum speed), and δ is the minimum spacing. Without loss of generality, we introduce variable buffer headway time, $h_f \geq 0$, to describe traffic in free-flow regime; therefore, headway in free-flow regime is $\tau + \delta/v_f + h_f$.

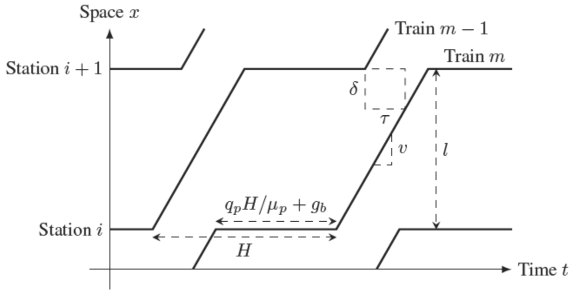
2.2 Steady State of Railway Operation

Here we consider a *steady state* of an urban rail transit operation under the aforementioned assumptions. A steady state is an idealized state of a traffic where the state (typically flow, density, and speed) is characterized by a *fundamental diagram (FD)* of the traffic (Daganzo, 1997). In the case of transit operation, it can be defined as a state such that: the model parameters, namely, $\mu_p, g_b, v_f, \tau, \delta$, are constant; the distance between each adjacent stations, l, is constant; the length of railroad is sufficiently long; all the trains stops every station; the headway time between each successive trains, H, is constant; the cruising speed, v, of all the trains is the same; and the passenger-flow to each station, q_p, is the same. Under a steady state, the dwell time of a train at a station is represented as $q_pH/\mu_p + g_b$; because n_p is equal to q_pH. Note that control strategy of the transit operation (e.g., schedule-based, headway-based) is not explicitly specified here, as in the Newell's car-following model (Newell, 2002). However, any strategy whose stationary point of its dynamics is the steady state should achieve the steady state, if there are no delays (Wada et al., 2012).

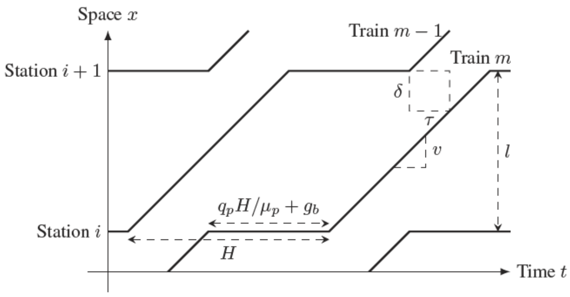
Railway operations under different steady states are illustrated by Fig. 1 as time–space diagrams, where the horizontal axis indicates time, the vertical axis indicates space, and the curves indicate trajectories of trains. In each sub-figure in the figure, train m arrives and departs station i, travels to station $i + 1$ with cruising speed v, and arrives station $i + 1$ under different conditions.



(**a**) Free-flowing regime: $v = v_f$, $h_f > 0$.



(**b**) Critical regime: $v = v_f$, $h_f = 0$.



(**c**) Congested regime: $v < v_f$, $h_f = 0$.

Figure 1. Time–space diagrams of rail transit operation under steady states.

2.3 Fundamental Diagram

Independent variables of traffic state of the transit operation can be defined as train-flow q, train-density k, and passenger-flow q_p. Now suppose that relation among the independent variables of traffic state under every steady state can be expressed using a function Q as follows:

$$q = Q(k, q_p). \tag{3}$$

The function Q can be defined as an FD of the transit operation. In fact, the function Q can be expressed as follows:

$$Q(k, q_p) = \begin{cases} \dfrac{lk - q_p/\mu_p}{g_b + l/v_f}, & \text{if } k < k^*(q_p), \\ -\dfrac{l\delta}{(l-\delta)g_b + \tau l}\left(k - k^*(q_p)\right) + q^*(q_p), & \text{if } k \geq k^*(q_p), \end{cases} \tag{4}$$

where $q^*(q_p)$ and $k^*(q_p)$ are train-flow and train-density, respectively, in a critical state with passenger-flow q_p; they are represented as

$$q^*(q_p) = \frac{1 - q_p/\mu_p}{g_b + \delta/v_f + \tau}, \tag{5}$$

$$k^*(q_p) = -\frac{(l-\delta)/v_f - \tau}{(g_b + \delta/v_f + \tau)\mu_p l} q_p + \frac{g_b + l/v_f}{(g_b + \delta/v_f + \tau)l}. \tag{6}$$

Due to the space limitation, the derivation of Eq. (4) cannot be provided here. However, one can confirm that Eqs. (3) and (4) are consistent with Edie's generalized definition of traffic states (Edie, 1963) of the steady state transit operation, namely,

$$q = \frac{1}{H}, \tag{7}$$

$$k = \frac{q_p H/\mu_p + g_b + l/v}{lH}, \tag{8}$$

$$\bar{v} = \frac{l}{q_p H/\mu_p + g_b + l/v}, \tag{9}$$

which are derived by applying the Edie's definition to the "minimum component of time–space diagram" of the steady state, which is a parallelogram-shaped area whose vertexes are time–space points of train m departs station i, train m arrives station $i+1$, train $m-1$ arrives station $i+1$, and train $m-1$ departs station i in Fig. 1.

2.4 Discussion

2.4.1 Features of Fundamental Diagram

The FD has following features which can be derived analytically from Eq. (4). Note that they can be easily found in the numerical example in Section 2.4.2.

The FD can be interpreted as a function determines train-flow q (and average-speed $\bar{v} = q/k$) under given train-density k, passenger-flow q_p (i.e., demand), and the technical parameters of the transit system (i.e., $\mu_p, g_b, v_f, \tau, \delta, l$). Although the FD equation (4) looks complicated, it represents simple relation: namely, piecewise linear (i.e., triangular) relation between q and k if q_p is fixed.

As mentioned, a traffic state of transit system can be categorized into three regimes: free-flowing, critical, and congested as in standard traffic flow theory. It means that there is a critical train-density $k^*(q_p)$ for given q_p. Then train traffic is in the free-flowing regime ($v = v_f, h_f > 0$) if $k < k^*(q_p)$, in the critical regime ($v = v_f, h_f = 0$) if $k = k^*(q_p)$, or in the congested regime ($v < v_f, h_f = 0$) otherwise. The congested regime can be considered as inefficient compared with the free-flowing regime; because the congested regime takes more time to transport the same amount passengers. The critical regime is the most efficient for travel time minimization, as well as passenger-crowding (i.e., number of passengers per a train, q_p/q) minimization. However, the critical regime requires more trains (i.e., higher train-density) than the free-flowing regime; therefore, it may not be the most efficient considering the operation cost.

Even in the critical regime, the average speed v is inversely proportional to passenger demand q_p. It means that travel time tends to increase if the passenger demand increases. In addition, size of the feasible area of (q, k) narrows as q_p increases. It means that operational flexibility of transit system declines as the passenger demand increased.

2.4.2 Numerical Example

A numerical example is shown in Fig. 2 where the parameter values used are: $u = 70$ (km/h), $\tau = 1/70$ (h), $\delta = 1$ (km), $\mu_p = 36\,000$ (pax/h), $g_b = 10/3600$ (h), $l = 3$ (km). In the figure, the horizontal axis represents train-density k, whereas the vertical axis represents train-flow q, and the plot color represents passenger-flow q_p . A slope of a straight line from a traffic state to the origin represents average speed v of the state.

The features discussed in Section 2.4.1 can be easily confirmed. For example, the figure can be read as follows. Suppose that the passenger demand per station is $q_p = 16\,000$ (pax/h) and have to be transported by the transit system. If number of trains in the system is corresponding to train-density

$k = 0.3$ (veh/km), then the resulting train traffic is with train-flow $q \simeq 15$ (veh/h) and average speed $\bar{v} \simeq 50$ (km/h). This is traffic state in the free-flowing regime. In addition, there is a congested state corresponding to a free-flowing state. The triangular $q-k$ relation mentioned before is clearly shown in the figure; the "left side" of the triangle corresponds to the free-flowing regime, the "top vertex" is corresponding to the critical regime, and the "right side" corresponds to the congested regime.

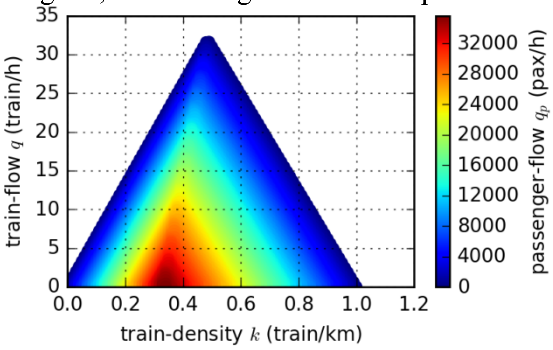


Figure 2. Numerical example of the FD.

2.4.3 Relation to Actual Railway System

Here we discuss relation between the proposed FD and actual transit system, as the proposed model and FD is based on simplified assumptions. The FD is fairly consistent with schedule-based train operation, although the FD does not consider a schedule explicitly. The FD can be considered as relation approximating an "average" operation schedule, if the schedule is designed to transport mass passenger demand (i.e., $v = v_f$ if $h_f > 0$ and $h_f = 0$ if $v \leq v_f$ hold). The "average" means it do not distinguish between non-express and express trains—this can be considered as limitation to some extent; however, it may not be essential as many transit systems in central metropolitan area (e.g., metro) do not operates express trains. The FD is fairly consistent with operation with adaptive control strategies in the real world, such as scheduled and/or headway-based control (Wada et al., 2012). This is because that aim of such control is usually to eliminate bunching—in other words, such control makes the operation steady. The FD ignores the passenger-crowding effect, as the passenger boarding model (1) is linear. Therefore, a regime with excessively large q_p and small q may not be consistent with actual transit system. This is a limitation of this model.

3. DYNAMIC MODEL BASED ON FUNDAMENTAL DIAGRAM

In this section, we formulate a model of urban rail transit operation where the demand (i.e., passenger-flow) and supply (i.e., train-flow) change dynamically. The proposed model is based on an *exit-flow model* (Carey and McCartney, 2004) of which the proposed FD is employed as the exit-flow function. In other words, the transit system is considered as an input-output system as shown in Fig. 3. The exit-flow modeling approach is often employed for area-wide traffic approximation and analysis using FD-like concepts (e.g., Daganzo, 2007; Geroliminis and Levinson, 2009).

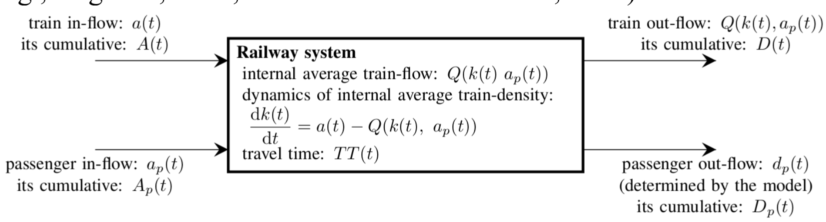


Figure 3. Railway system as input-output system.

3.1 Formulation

Let $a(t)$ be in-flow of trains to the transit system, $a_p(t)$ be in-flow of passengers, $d(t)$ be out-flow of trains from the transit system, and $d_p(t)$ be out-flow of passengers, at time t, respectively. Without loss of generality, we assume that the initial time is 0, and therefore $t \geq 0$ holds. Let $A(t), A_p(t), D(t)$, and $D_p(t)$ be cumulative numbers of $a(t), a_p(t), d(t)$, and $d_p(t)$, respectively (e.g., $A(t) = \int_0^t a(s)\mathrm{d}s$). Let $TT(t)$ be the travel time of a train (and a passenger) who entered the system at time t, and its initial value $TT(0)$ be given as free-flow travel time under $q = a(0), q_p = a_p(0)$. In order to simplify the formulation, the trip length of the passengers is assumed to be equal to that of trains. It means that TT is travel time of both of the trains and passengers. Note that, in reality, the $a(\cdot)$ and $a_p(\cdot)$ will be determined by transit operation plan and passenger departure time choice, respectively. The $d(\cdot), d_p(\cdot)$, and $TT(\cdot)$ are endogenously determined by the proposed model.

The train traffic can be calculated as follows. First, in accordance with the manner of the exit-flow modeling, the exit-flow, $d(t)$, is assumed as $d = Q\left(k(t), a_p(t)\right)$, where the FD function, $Q(\cdot)$, is considered as an exit-flow function. It means that the dynamics of the transit system is modeled as

$$\frac{\mathrm{d}k(t)}{\mathrm{d}t} = a(t) - Q\left(k(t) - a_p(t)\right), \tag{10}$$

by taking a conservation of trains into account. Note that, the average train-density $k(t)$ is, by the definition, derived as

$$k(t) = \frac{A(t) - D(t)}{L}, \tag{11}$$

where L can be interpreted as trip length of trains (e.g., length of the transit route). Then, $d(t)$ and therefore $D(t)$ can be sequentially calculated using $TT(0), a(t), a_p(t)$, and Eqs. (10) and (11).

The passenger traffic can be calculated as follows. By the definition of travel time of trains, $A(t) = D\big(t + TT(t)\big)$ holds. Therefore, since A and D are already obtained, the travel time $TT(t)$ such that $A(t) = D\big(t + TT(t)\big)$ holds can be calculated. Then, $D_p(t)$ and therefore $d_p(t)$ can be calculated by the definition of travel time of passengers, which is $TT(t)$ as well: $A_p(t) = D_p\big(t + TT(t)\big)$.

3.2 Discussion

The proposed macroscopic model calculate train out-flow $d(t)$ and passenger out-flow $d_p(t)$, based on the FD function $Q(\cdot)$ and initial and boundary conditions $a(t)$, $a_p(t)$, and $TT(0)$. Therefore, it can derive social cost and benefit of a transit system.

The notable feature of the model is its high tractability; as it is based on an exit-flow model. Therefore, we expect that management strategies (e.g., dynamic pricing in morning commute) in transit systems can be analyzed by using the proposed model.

The proposed model would approximate macroscopic behavior of a transit operation well if one with high-frequency operation (i.e., small headway time), densely spaced stations, and spatially homogeneous passenger demand are considered. A typical example is a subway in central metropolitan area. This is because an exit-flow model can be reasonable if change of in-flow is moderate compared with the relaxation time of the dynamical system. Note that, such transit system is worth investigating; because congestion and delay are often found during rush hours due to the excessive demand in such transit systems.

4. VALIDATION

In this section, we validate quantitative behavior of the macroscopic model by comparing its results with that of the microscopic model (i.e., Eqs. (1) and (2)) with the same parameter values. The validity of the macroscopic model can be investigated by comparing the output with that of microscopic models. Specifically, computed D where the same A and A_p are given is to be compared.

4.1 Simulation Setting

Following situation is considered. The parameters of the transit operation are set as the same to those of Section 2.4.2, and are shared by the both of microscopic and macroscopic models. Railroad is considered as a one-way corridor. The stations are equally spaced with interval l, and the total number is 10. Trains enter the railroad with flow $a(t)$; in the microscopic model, a discrete train enters the railroad from the upstream boundary station if integer part of $A(t)$ is incremented. In the microscopic model, the trains leave the railroad from the downstream boundary station without restriction other than the passenger boarding. Passengers arrive each station with flow $a_p(t)$. The simulation duration is 4 hours.

The microscopic model without any control is asymptotically unstable as proven by Wada et al. (2012); it means that dynamically changing demand and supply shall cause train bunching, making the experiment unrealistic and useless. Therefore, a control strategy such that a stable equilibrium of its dynamics is identical to the underlying steady state is implemented to the microscopic model. It is a headway-based control using two control measures: dwelling time extension and free-flow speed increase (up to 80 km/h in this experiment). The implemented control can be considered as realistic; as similar operation is executed in practice. See Wada et al. (2012) for the details.

The $a(t)$ and $a_p(t)$ are exogenously given to mimic morning commute situation, namely, so-called S-shaped $A(t)$ and $A_p(t)$ are considered. The minimum and maximum values are: $a(0) = a(4) = 10$ (veh/h), $a(2) = 15$ (veh/h), $a_p(0) = a_p(4) = 0.1\mu_p$ (pax/h), and $a_p(2) = 0.5\mu_p$ (pax/h). The values for $t \in (0,2), (2,4)$ are given by linear interpolation. It means that the peak time is $t = 2$ (h). They are shared by the both of microscopic and macroscopic models.

4.2 Results

A result of the microscopic model is shown in Fig. 4 as a time–space diagram. The colored curves represent trajectories of each train that travels upward direction while stopping at every station. Around the peak time period ($t = 2$), train congestion is observed; namely, some of the trains stop occasionally between stations in order to keep the safety interval. The congestion is due to the excessive passenger demand; therefore, a situation during rush hour is reproduced well.

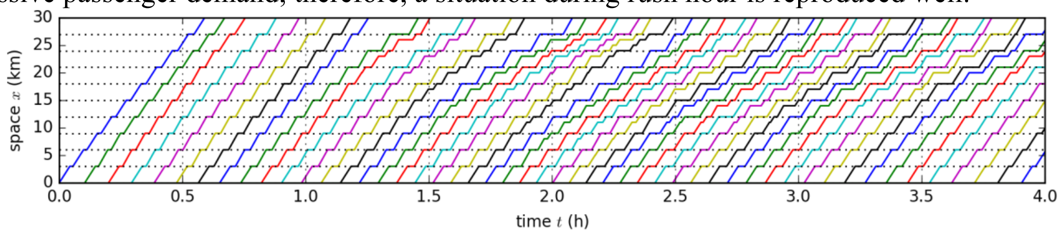


Figure 4. Result of the microscopic model.

A result of the macroscopic model is shown in Fig. 5 as cumulative plots. Fig. 5a shows the cumulative curves of trains, where the blue curve represents the in-flow A and the red one represents the out-flow D. Fig. 5b shows those of passengers in the same manner. Congestion and delay can be found around the peak period (it is more remarkable in the passenger traffic). For example, during the peak time period, $d_p(t)$ is smaller than $a_p(t)$ and $a_p(t')$ ($t = t' + TT(t')$). It means that the throughput of the transit system is reduced due to the passenger demand.

Comparison between the macroscopic and microscopic models is shown in Fig. 6 in terms of the cumulative plot of train. In the figure, the solid curves represent the results of the macroscopic model; therefore, they are identical to Fig. 5b. The dots represent that of the microscopic model. According to the figure, the macroscopic model reproduced results of the microscopic one fairly precisely. For example, congestion and delay during the peak time period were captured well.

5. CONCLUSION

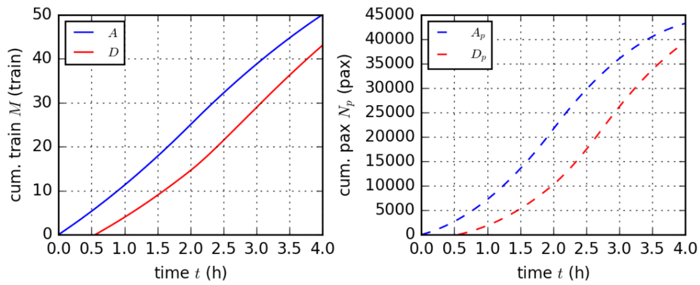
In this paper, the following three models of urban rail transit system are analyzed:

- *Microscopic model*: A model describing trajectory of individual train and passengers based on the Newell's car-following model and passenger boarding model. It can be solved by simulation.
- *Fundamental diagram*: Relation among train-flow, train-density, and passenger-flow of the microscopic model under a steady state. It is a closed-form equation.

- *Macroscopic model*: A model describing train and passenger traffic using an exit-flow model whose exit-flow function is the FD. It can be solved by simple simulation.

The FD and macroscopic model are the original works developed by this study, while the microscopic model has been proposed by Wada et al. (2012). The microscopic model can be considered as a lower order approximation of actual transit system. The FD represents the exact relation among steady state traffic variables in the microscopic model. The macroscopic model can be considered as an FD-based macroscopic approximation of area-wide behavior of the microscopic model. Qualitative features of the FD agree with actual phenomena to some extent. In addition, according to the results of the numerical experiment, the macroscopic model can reproduce behavior of the microscopic model.

The proposed FD and macroscopic model have simplicity, high mathematical tractability, and good approximation accuracy. Therefore, we expect that they will contribute for obtaining policy implications on management strategies of rail transit systems (e.g., pricing and control for morning commute problems) by applying the models to dynamic traffic assignment problems.



(a) Train **(b)** Passenger

Figure 5. Result of the microscopic model.

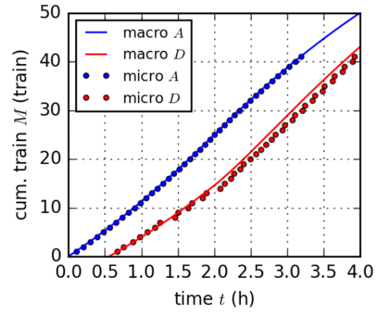


Figure 6. Comparison between the macroscopic and microscopic models

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