

# **Valuation of Credit Risk from Firm's Profit under Hyperbolic Sine Process**

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200720802

(Master's Program in Business Administration and Public Policy)

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Submitted to the Graduate School of  
Systems and Information Engineering  
in Partial Fulfillment of the Requirements  
for the Degree of Master of Business Administration  
The University of Tsukuba

March 2009

# Abstract

Mathematical credit risk models in the literature mainly base on Brownian motion although it is known that actual financial data provides a different statistical behavior than that implied by these models. Hyperbolic Sine process proposed by Takahashi (2007) is an appropriate tool to increase accuracy of models in finance. It can be used to model stock prices, and term structures of interest rates, thus allowing more accurate derivative pricing and risk management. In this paper, we build a model that takes the firm's profit as underlying in an option approach to value credit risk and discuss how general Hyperbolic Sine process can be applied to this new credit risk model. We mainly deal with the question of estimating default probabilities from our model in the valuation of credit risk. Through empirical tests, we get results which are consistent with the credit ratings published by respected rating agencies. Besides, this paper establishes a new relationship between stock price and firm's profit in an option approach which will be useful in making investment strategies in the future.

**KEY WORDS:** Structural credit model; Hyperbolic Sine process; Profit option model; Risk neutral measure; Default probability; Implied volatility

# Contents

|   |           |
|---|-----------|
| <b>1. Introduction .....</b>  | <b>1</b>  |
| <b>2. Classical structural credit model.....</b>                              | <b>3</b>  |
| <b>3. Hyperbolic Sine process and its application in option pricing .....</b> | <b>6</b>  |
| 3.1 Hyperbolic Sine process.....  | 6         |
| 3.2 Risk neutral measure .....  | 7         |
| 3.3 European option valuation .....   | 8         |
| <b>4. Profit option model.....</b>  | <b>10</b> |
| 4.1 Distribution of firm's profit.....  | 10        |
| 4.2 Profit option model.....  | 12        |
| <b>5. Empirical tests.....</b>  | <b>16</b> |
| 5.1 Data .....  | 16        |
| 5.2 Parameters .....  | 16        |
| 5.3 Solution of implied volatility of standardized profit.....                | 17        |
| 5.4 Credit Ratings.....   | 19        |
| 5.5 Testing results .....   | 20        |
| <b>6. Conclusions and future research.....</b>                                | <b>22</b> |
| 6.1 Conclusions .....   | 22        |
| 6.2 Future research .....   | 23        |
| <b>Acknowledgements .....</b>   | <b>24</b> |
| <b>Bibliography .....</b>   | <b>25</b> |
| <b>Appendix: Solution of two reletive stochastic process.....</b>             | <b>28</b> |