

Nonparametric Volatility Estimation of the Japanese
Stock Price Index

Katsuyuki Takahashi
200620872

(Master's Program in Business Administration and Public Policy)

Advised by Isao Shoji

Submitted to the Graduate School of
Systems and Information Engineering
in Partial Fulfillment of the Requirements
for the Degree of Master of Business administration
at the
University of Tsukuba

January 2008

日本の株価指数のノンパラメトリックボラティリティ推定

Abstract

The purpose of this article is to estimate the volatility function of TOPIX index. TOPIX index represents Japanese main stock prices. Then the process of the index is supposed to follow a general continuous-time diffusion model. A volatility function is defined as the square of a diffusion function here. In recent years, we can gain high-frequency data involving stock markets. Hence, this supposition is more effective. Some nonparametric techniques have been studied to estimate a volatility function. A local linear approach is, however, useful compared with the others in practical estimation. In this paper, some simulation experiments show that the local linear method is much more effective than a semiparametric approach developed by Aït-Sahalia (1996). Based on a local linear approach, the volatility function of TOPIX index has been estimated, calculated using samples of different periods. Estimated volatility functions have seemed to depend on the period of used samples. This arrives at the idea that the volatility function varies with the period of time. An approximate confidence interval indicates a part of evidence that these estimated functions are different from each other significantly. Besides, the relation between the volatility function and the trend of TOPIX index is an interesting subject.

KEY WORDS: Continuous-time diffusion model; High-frequency data; Local linear kernel regression; Stock price index; Volatility function;

Contents

1	Introduction	1
2	Estimation methods	5
2.1	Semiparametric estimation	5
2.1.1	The semiparametric estimator of $\sigma^2(X_t)$	5
2.1.2	The parametric estimator of $\mu(X_t)$	6
2.1.3	Kernel density estimation	8
2.2	The local linear estimator of $\sigma^2(X_t)$	9
2.2.1	Volatility approximation	9
2.2.2	Kernel regression	10
2.2.3	Properties of the local polynomial estimator	13
3	Simulations	15
4	Volatility estimation of TOPIX index	20
4.1	data description	21
4.2	Using subsamples of different periods	22
4.3	Further analysis	28
5	Conclusion	32