

# Improving the Kim's Approach for Pricing American Put Option

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アメリカンプットオプションの計算方法に対して－Kimのアプローチの改善

# Abstract

There exist some formulas to calculate the American put option in the world, and non-trivial computations are required to solve it. From the beginning of the 1990s, there has been an increase of pricing theories for the valuation of the American put option. The properties associated with the optimal exercise boundary with a step function are noticed, and numerical techniques to implement the valuation formulas are presented.

In this thesis, based on the optimal exercise boundary theory, we propose to price an American put option by approximating its early exercise boundary as a piece-wise linear function. Closed form formulas are obtained in terms of the continuation of the piece-wise linear function. It is demonstrated that a piece-wise linear function of the optimal exercise boundary has the accuracy to fulfill the smooth pasting condition. In the conclusion, we improve the original Kim's (1990) method to price the American put option.

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